Q. P. Code: 21236

Time: 3 Hours **Marks: 80**

Note: 1) Q.1 is **COMPULSORY.**

2) Attempt **ANY 3 questions** from Q.2 to Q.6

3) Use of scientific calculators allowed.

4) Figures to right indicate marks.

Q.1 **a)** Find the Laplace transform of $e^{-2t}\, t\, cost$ **(05)**

b) Find the inverse Laplace transform of $\frac{3s+7}{s^2-2s-3}$ **(05)**

c) Determine whether the function f(z) = $(x^3 + 3xy^2 - 3x) + i(3x^2y - y^3 + 3y)$ is analytic and if so find its derivative. **(05)**

d) Find the Fourier series for f(x) = x^2 in the interval (-π, π). **(05)**

Q.2 a) Evaluate $\int_0^\infty \left(\frac{sin2t + sin3t}{t\, e^t}\right) dt = \frac{3\pi}{4}$ **(06)**

b) Find the Z- Transform of $\left\{\left(\frac{1}{4}\right)^{|k|}\right\}$ **(06)**

c) Show that the function v= $e^x(x\, siny + y\, cosy)$ is a harmonic function. Find its harmonic conjugate and corresponding analytic function. **(08)**

Q.3 a) From 8 observations the following results were obtained. **(06)**

$\sum x = 59$; $\sum y = 40$; $\sum x^2 = 524$; $\sum y^2 = 256$; $\sum xy = 364$.

Find the equation of the line of regression of x on y and the coefficient of correlation.

b) Find the bilinear transformation which maps the points z = -1 , 0 , 1 onto the points w = -1, -*i*, 1. **(06)**

c) Obtain half-range sine series for f(x) =$(x-1)^2$ in 0 < x < 1 .
Hence find $\sum_{n=1}^{\infty} \frac{1}{n^2}$ **(08)**

Q.4 a) Find the inverse Laplace Transform by using convolution theorem $\frac{1}{(s^2+a^2)(s^2+b^2)}$ **(06)**

b) Compute Spearman's Rank correlation coefficient for the following data: **(06)**

X	85	74	85	50	65	78	74	60	74	90
Y	78	91	78	58	60	72	80	55	68	70

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c) Find the inverse Z-transform for the following; **(08)**

i) $\frac{1}{(z-5)^2}$, $|z| < 5$ ii) $\frac{z}{(z-2)(z-3)}$, $|z| > 3$

Q.5 a) Using Laplace Transform evaluate $\int_0^\infty e^{-t}\,(1+3t+t^2)\,H(t-2)\,dt$ **(06)**

b) Prove that $f_1(x) = 1$; $f_2(x) = x$; $f_3(x) = (\frac{3x^2-1}{2})$ are orthogonal over (-1, 1). **(06)**

c) Solve using Laplace transform $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2\,e^{3x}$, $y = 2$, $y' = 3$ at $x = 0$. **(08)**

Q.6 a) Find the complex form of Fourier series for $f(x) = e^x$, $(-\pi, \pi)$. **(06)**

b) If u , v are harmonic conjugate functions, show that uv is a harmonic function. **(06)**

c) Fit a straight line of the form y = a + bx to the following data and estimate the value of y for x = 3.5 **(08)**

x	0	1	2	3	4
Y	1	1.8	3.3	4.5	6.3

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